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LINEARIZATION OF HOLOMORPHIC MAPPINGS ON $C(K)$-SPACES

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ABSTRACT

We prove a universal mapping theorem for "integral" holomorphic mappings on the open unit ball of $C(K)$. In our theorem, the universal space is $C(K)$, and the universal mapping is increasing in the positive cone of $C(K)$.

Introduction

Universal mapping theorems for holomorphic mappings on Banach or locally convex spaces have been presented, for example, in [Ma], [Mu1], [Mu2], [Mu–N] and [G–G–M]; see also [R] and [S]. Given an open subset U of a Banach space X and a Banach space Y, a typical universal mapping result gives a universal Banach space Z and a universal holomorphic mapping $\psi: U \to Z$ such that, for every holomorphic mapping (maybe: of some special type) $F: U \to Y$, there exists a bounded linear operator $B_F: Z \to Y$ with the property that $F = B_F \circ \psi$. Usually, the structure of the universal Banach space remains quite unclear.

In this paper we study a more restricted situation, namely "integral" (to be explained later) holomorphic mappings on the open unit ball U of the Banach space $C(K)$. We prove a universal mapping theorem having the special property that the universal Banach space is $C(K)$. Moreover, our universal mapping is increasing (with respect to the natural positive cone of $C(K)$).

For the proofs we use the technique developed in [T1]. We need special continuous surjections φ between compact metric spaces K and K_1, which have the property that the pull-back operator $\varphi^\circ: C(K_1) \to C(K)$, $\varphi^\circ f = f \circ \varphi$,

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admits a bounded left inverse. Thus our result gives a new application of the theory of averaging operators (see below for terminology).

1. Notation. Integral holomorphic mappings

We denote by $\mathbb{N}$ the set $\{1, 2, 3, \ldots\}$. All Banach spaces are over the complex scalar field. The space of bounded linear operators between the Banach spaces X and Y is denoted by $L(X, Y)$, or by $L(X)$, if $X = Y$; the dual of X is denoted by X^*. The absolutely convex hull of a subset A of a Banach space is denoted by $\Gamma(A)$.

For general topology we refer to [Ku]. If K is a compact metric space, we denote by $C(K)$ the Banach space of continuous, complex valued mappings, endowed with the sup-norm. If K_1 and K_2 are compact metric spaces and φ: $K_1 \to K_2$ is a continuous surjection, we denote by φ° the linear isometry from $C(K_2)$ into $C(K_1)$ given by $\varphi^\circ f = f \circ \varphi$. If $\varphi^\circ(C(K_2))$ is 1-complemented in $C(K_1)$, i.e., if there exists a contractive projection from $C(K_1)$ onto $\varphi^\circ(C(K_2))$, we say that φ admits a regular averaging operator. (Note that in this case the map φ° also has a contractive left inverse.) For more details we recommend the reference [L–T], Sections II.4.h,i, and [P].

For complex analysis in infinite-dimensional spaces we refer to [D2] and [C]. If X and Y are Banach spaces and $n \in \mathbb{N}$, we denote by $\mathcal{P}(^n X, Y)$ the space of continuous n-homogeneous polynomials $X \to Y$.

Recall that a continuous n-linear form F on $C(K)^n$ is called integral, if there exists a $\mu(F) \in C(K^n)^*$ such that

$$F(f_1, \ldots, f_n) = \left\langle \prod_{k=1}^{n} f_k \circ \pi_n^{(k)}, \mu(F) \right\rangle, \tag{1.1}$$

where $f_k \in C(K)$, $\pi_n^{(k)}$ is the canonical projection from K^n onto the kth coordinate space and the product on the right-hand side is taken pointwise. In particular, every continuous linear form is integral.

Let $U \subset C(K)$ be open and F: $U \to Y$ holomorphic. We write the Taylor series of F at the point $y \in U$ as

$$F(x) = F_0 + \sum_{n=1}^{\infty} F_n^{(y)}(x - y), \tag{1.2}$$

where $F_0 \in Y$ and $F_n^{(y)} \in \mathcal{P}(^n C(K), Y)$; we denote by $\hat{F}_n^{(y)}$ the corresponding symmetric n-linear mapping. (If $y = 0$, we omit the superindex (y).)

1.1 Definition: Let Y be a Banach space, let $U \subset C(K)$ be open, let $F: U \to Y$ be a holomorphic mapping, let $B \subset U$ be an open ball with center y and radius r, and let $S \subset Y^*$ be a bounded subset. We say that F is uniformly (S, B)-integral, if

1° for every $t \in S$, $n \in \mathbb{N}$, the n-linear form

$$(f_1, \dots, f_n) \mapsto \langle \hat{F}_n^{(y)}(f_1, \dots, f_n), t\rangle$$

is integral (write $\mu(F, n, t)$ for the corresponding element of $C(K^n)^*$ as in (1.1)),

2°

$$(1.3) \qquad ||F||_{S,B} := \sup_{t\in S}\Big\{|\langle F_0, t\rangle| + \sum_{n=1}^{\infty} r^n ||\mu(F, n, t)||_{C(K^n)^*}\Big\} < \infty,$$

and

3° the mapping

$$(1.4) \qquad t \mapsto \sum_{n=1}^{\infty} \langle h_n, \mu(F, n, t)\rangle r^n$$

is, for arbitrary $h_n \in C(K^n)$ with $||h_n|| \leq 1$, continuous $S \to \mathbb{C}$, when S is endowed with the weak* topology.

Conditions 1° and 3° are natural generalizations of the conditions appearing in Theorem 2.a) of [T1]. The condition 2° is a technical requirement necessary for our purposes.

Motivation for Definition 1.1 is provided by the norm equality in Theorem 2.1. Moreover, Definition 1.1 and Theorem 2.1 are needed in an essential way in [T2] to prove other representation results for holomorphic mappings.

We remark that the above concept of integral holomorphic mappings does not coincide with the definition of mappings of integral holomorphy type in [D1] and [A]. Nevertheless, our definition is quite natural and gives quite a large class of holomorphic mappings, as shown by the following

1.2 Examples: (See also Proposition 1.3.2°.) 1° For all $n \in \mathbb{N}$ the homogeneous polynomials P_n: $C(K) \to C(K)$, $f \mapsto f^n$ (pointwise multiplication) are uniformly (S, B)-integral for every S and B as in Definition 1.1. (The symmetric n-linear mapping corresponding to P_n is $\hat{P}_n \in L(C(K)^n, C(K))$, $\hat{P}_n$: $(f_1, \dots, f_n) \mapsto$

$\prod_{k=1}^{n} f_k$. Given $t \in S$, the element $\mu(P_n, n, t)$ is $D't$, where D' is the transpose of the diagonal operator $D \in L(C(K^n), C(K))$, $(Dh)(s) = h(s, \ldots, s)$.)

Note especially that *the identity operator is uniformly integral, but that it is not e.g. a Pietsch-integral operator* (see Theorem VI.3.12 in [D–U]). Hence, the classical definitions of integral mappings do not seem so useful for our purposes; they give too restricted classes of examples.

2° Let $U \subset C(K)$ be the open unit ball, let $Y = C(K)$ and let h be a scalar valued holomorphic mapping on the open unit disc of $\mathbb{C}$ such that its Taylor coefficients at 0 form an absolutely summable sequence. Then the map $(Hf)(t) = h(f)(t)$, $f \in U$, $t \in K$, is uniformly (K, U)-integral on U. (Here the set K is identified in the canonical way with a subset of $C(K)^*$. The elements $\mu(H, n, t)$ of Definition 1.1 are Dirac measures multiplied by complex scalars coming from the Taylor series of h.)

3° Denote by $U \subset C(K)$ the open unit ball. If $F_n \colon K \times K^n \to \mathbb{C}$ is for all $n \in \mathbb{N} \cup \{0\}$ a continuous function satisfying

$$\sum_{n=0}^{\infty} ||F_n||_{C(K^{n+1})} < \infty,$$

then the holomorphic mapping

$$f \mapsto \sum_{n=0}^{\infty} \int_{K^n} F_n(\cdot, s_1, \ldots, s_n) f(s_1) \cdots f(s_n) d\mu_n(s), \tag{1.5}$$

where $s = (s_1, \ldots, s_n) \in K^n$ and μ_n is a Radon probability measure on K^n, is uniformly (K, U)-integral $U \to C(K)$.

If h is as in 2° and U, F_1 and μ_1 are as above, then also the map

$$f \mapsto \int_K F_1(\cdot, s) h(f(s)) d\mu_1(s) \tag{1.6}$$

is uniformly (K, U)-integral.

Using the notation of Definition 1.1 we now state

1.3 Proposition: 1° *If S satisfies, for some $c > 0$, $\sup_{t \in S} |\langle f, t\rangle| \geq c||f||_Y$ for all $f \in Y$, then the space of uniformly (S, B)-integral mappings $B \to Y$ is a Banach space.*

2° *If* $F\colon B \to Y$ *is uniformly* (S, B)*-integral, the center of* B *is* 0, *and* $A \in L(C(K))$ *is a contraction, then* $F \circ A$ *is uniformly* (S, B)*-integral.*

3° *If* $S_1 \subset S \subset Y^*$ *and* $B_1 \subset B$ *is an open ball, and if* $F\colon B \to Y$ *is uniformly* (S, B)*-integral, then* F *is uniformly* (S_1, B_1)*-integral.*

Proof: 1° We prove only the completeness. We may assume that the center of B is 0. If $G\colon B \to Y$ is uniformly (S, B)-integral, a straightforward estimate, using the assumption on the set S, shows that

$$||G||_0 := \sup_{f\in B} ||G(f)||_Y \leq c^{-1}||G||_{S,B}. \tag{1.7}$$

Let now $(F^{(m)})_{m=1}^{\infty}$ be a $||\cdot||_{S,B}$-Cauchy sequence of uniformly (S, B)-integral mappings $B \to Y$. By (1.7), $(F^{(m)})_{m=1}^{\infty}$ is Cauchy also with respect to $||\cdot||_0$. Since the space $H^\infty(B; Y)$ of bounded holomorphic mappings $B \to Y$ is complete with respect to the sup-norm ($= ||\cdot||_0$), we see that $(F^{(m)})_{m=1}^{\infty}$ converges in $||\cdot||_0$ to a holomorphic map $F\colon B \to Y$.

Write the Taylor series $F = \sum_n F_n$ and $F^{(m)} = \sum_n F_{n,m}$.

Let us fix $n \in \mathbb{N}$ and $t \in S$ for a moment. The sequence $(\mu(F^{(m)}, n, t))_{m=1}^{\infty}$ converges in $C(K^n)^*$ to an element $\mu(n, t)$, in view of (1.3). For all m and $f \in B$ we have the equality

$$\langle F_{n,m}(f), t\rangle = \left\langle \prod_{k=1}^{n} f \circ \pi_n^{(k)}, \mu(F^{(m)}, n, t)\right\rangle,$$

where $\pi_n^{(k)}$ is as in (1.1). Since the subspace of n-homogeneous polynomials is complemented in $H^\infty(B; Y)$ (as a consequence of the Cauchy integral formula), we have $F_{n,m} \to F_n$ in the sup-norm, and hence

$$\langle F_n(f), t\rangle = \left\langle \prod_{k=1}^{n} f \circ \pi_n^{(k)}, \mu(n, t)\right\rangle.$$

This implies (in view of the 1–1 correspondence of homogeneous polynomials and symmetric multilinear mappings; $\mu(n, t)$ is symmetric with respect to the coordinate changes in $C(K^n)$)

$$\langle \hat{F}_n(f_1, \ldots, f_n), t\rangle = \left\langle \prod_{k=1}^{n} f_k \circ \pi_n^{(k)}, \mu(n, t)\right\rangle \tag{1.8}$$

for $f_k \in B$. This proves 1° of Definition 1.1 for F.

To see 2° of Definition 1.1, we fix $t \in S$ and consider for every $m \in \mathbb{N}$ the sequence

$$M^{(m)}(t) := \Big(\langle F_{0,m}, t\rangle, r\mu(F^{(m)}, 1, t), r^2\mu(F^{(m)}, 2, t), \ \ldots\Big) \in \ell_1((C(K^n)^*)_{n=0}^\infty).$$

Since $(F^{(m)})_{m=1}^\infty$ is Cauchy with respect to (1.3), the sequence $(M^{(m)}(t))_{m=1}^\infty$ is Cauchy in $\ell_1((C(K^n)^*)_n)$ and converges to $M(t) \in \ell_1((C(K^n)^*)_n)$. In view of (1.3) the convergence is even uniform with respect to t. Hence,

$$\sup_{t\in S} ||M(t)||_{\ell_1((C(K^n)^*)_n)} < \infty. \tag{1.9}$$

But the nth coordinate of $M(t)$ equals $r^n\mu(n,t)$ above. This, (1.8) and (1.9) imply 2° of Definition 1.1 for F.

For all $n \in \mathbb{N}$, let $h_n \in C(K^n)$, $||h_n|| \leq 1$. By assumption, for every m, the map

$$F_*^{(m)}: t \mapsto \sum_{n=1}^{\infty} \langle h_n, \mu(F^{(m)}, n, t)\rangle r^n$$

is continuous $S \to \mathbb{C}$ for the weak*-topology of S. Since the sequence $(F^{(m)})_{m=1}^\infty$ is Cauchy with respect to the norm $||\cdot||_{S,B}$, the sequence $(F_*^{(m)})_{m=1}^\infty$ is Cauchy in $C(S)$. This implies that F satisfies 3° of Definition 1.1.

2° Recall that if $A \in L(C(K))$ and $||A|| \leq 1$, then for all $n \in \mathbb{N}$, $n \geq 2$, the map

$$A \otimes_\varepsilon \cdots \otimes_\varepsilon A: \prod_{k=1}^{n} f_k \circ \pi_n^{(k)} \mapsto \prod_{k=1}^{n} Af_k \circ \pi_n^{(k)},$$

where $f_k \in C(K)$ and $\pi_n^{(k)}$ is as in (1.1), can be extended to a linear contraction $A_n \in L(C(K^n))$ (see e.g. [Kö], 44.7.(3) and 44.4.(1)). If now F is as in the assumption and $\mu(F, n, t) \in C(K^n)^*$ is for all n, t as in Definition 1.1, then we can define

$$\mu(F \circ A, n, t) := A_n'\mu(F, n, t),$$

where $A_n' \in L(C(K^n)^*)$ is the transpose of A_n. It is a straightforward matter to verify that the conditions of Definition 1.1 are satisfied.

3° Let z and ϱ be the center and radius of B_1 so that $||z - y|| \leq r - \varrho$. If $k, n \in \mathbb{N}$ and $n \leq k$, we denote by $\pi_k^{(n,k)}: K^k \to K^n$ the canonical projection onto the last n coordinate spaces. We then define, for all $n \in \mathbb{N}$, $t \in S_1$, the linear form $\mu^{(z)}(F,n,t) \in C(K^n)^*$ by

(1.10)
$$\langle h, \mu^{(z)}(F,n,t)\rangle := \sum_{k=n}^{\infty} \binom{k}{n} \left\langle \left(\prod_{m=1}^{k-n} (z-y)\circ \pi_k^{(m)}\right) \left(h \circ \pi_k^{(n,k)}\right), \mu(F,k,t) \right\rangle.$$

(Here $\prod_{m=1}^{0}(z-y)\circ\pi_k^{(m)}$ is understood to be the constant 1.) The convergence of (1.10) follows for every $t \in S_1$ by the estimate

(1.11)
$$\begin{aligned}
&\sum_{n=1}^{\infty} ||\mu^{(z)}(F,n,t)||_{C(K^n)^*}\varrho^n \\
&\le \sum_{n=1}^{\infty}\sum_{k=n}^{\infty} \binom{k}{n} \sup_{\substack{h\in C(K^n),\\ ||h||\le 1}} |\langle (\prod_{m=1}^{k-n} (z-y)\circ\pi_k^{(m)})(h\circ\pi_k^{(n,k)}), \mu(F,k,t)\rangle|\varrho^n \\
&\le \sum_{k=1}^{\infty}\sum_{n=1}^{k} \binom{k}{n} ||\mu(F,k,t)||_{C(K^k)^*}\ ||z-y||^{k-n}\varrho^n \\
&\le \sum_{k=1}^{\infty} ||\mu(F,k,t)||_{C(K^k)^*} \sum_{n=1}^{k}\binom{k}{n}(r-\varrho)^{k-n}\varrho^n \\
&\le \sum_{k=1}^{\infty} ||\mu(F,k,t)||_{C(K^k)^*} r^k < \infty.
\end{aligned}$$

By [C], Theorem 11.11 and (1.10) above, (1.11) also shows that 1°–2° of Definition 1.1 hold for S_1 and B_1 instead of S and B.

Finally, to see 3° of Definition 1.1, assume that $h_n \in K^n$, $||h_n|| \le 1$ for all n. We define, for all $k \in \mathbb{N}$,

$$\eta_k := r^{-k}\sum_{n=1}^{k} \varrho^n \binom{k}{n} \left(\prod_{m=1}^{k-n}(z-y)\circ\pi_k^{(m)}\right)\left(h_n \circ \pi_k^{(n,k)}\right) \in C(K^k).$$

We have

$$\begin{aligned}
||\eta_k||_{C(K^k)} &\le r^{-k}\sum_{n=1}^{k}\binom{k}{n}\varrho^n ||z-y||^{k-n} \\
&\le r^{-k}\sum_{n=1}^{k}\binom{k}{n}\varrho^n (r-\varrho)^{k-n} \le 1.
\end{aligned}$$

It follows from (1.10) that for all $t \in S_1$

$$\sum_{n=1}^{\infty}\langle h_n, \mu^{(z)}(F,n,t)\rangle\varrho^n = \sum_{k=1}^{\infty}\langle \eta_k, \mu(F,k,t)\rangle r^k.$$

Hence, 3° for the case S_1, B_1 follows from 3° for the case S, B. ∎

2. A universal mapping theorem

In this section we prove a universal mapping theorem for (K, U)-integral holomorphic mappings on the open unit ball of $C(K)$. The point is that in our theorem the space $C(K)$ itself can be taken as the universal Banach space.

In Theorem 2.1 we denote by X a Banach space, by K be an uncountable compact metric space, by U the open unit ball of $C(K)$ and by S the closed unit ball of X^*. The set K is also considered as a subset of $C(K)^*$: for every $t \in K$ there corresponds the point evaluation δ_t: $f \mapsto f(t)$, $f \in C(K)$. This identification is a homeomorphism, when $C(K)^*$ is endowed with the weak*-topology. We denote by ι: $X \hookrightarrow X^{**}$ the canonical embedding.

2.1. THEOREM: *There exists a universal holomorphic mapping ψ: $U \to C(K)$ such that for every uniformly (K, U)-integral (resp. (S, U)-integral) holomorphic F: $U \to C(K)$ (resp. $U \to X$) there exists $B_F \in L(C(K))$ (resp. $B_F \in L(C(K), X^{**})$) such that the following diagram commutes:*

$$\begin{array}{ccc} U & \xrightarrow{\;F\;} & C(K) \\ & {\scriptstyle\psi}\searrow \quad \nearrow{\scriptstyle B_F} & \\ & C(K) & \end{array} \qquad \left(\text{resp.} \quad \begin{array}{ccccc} U & \xrightarrow{\;F\;} & X & \xhookrightarrow{\;\iota\;} & X^{**} \\ & {\scriptstyle\psi}\searrow & & \nearrow{\scriptstyle B_F} & \\ & & C(K) & & \end{array} \right).$$

Moreover, $||F||_{K,U} = ||B_F||$ (resp. $||F||_{S,U} = ||B_F||$).

Proof: 1° We prove the existence of the universal mapping. Let us denote by Q the Hilbert cube $\prod_{n=1}^{\infty}[0, 1]$. Since every compact metric space can be embedded in Q (see e.g [Ku] II.22.II), it is possible to find a sequence $(H_n)_{n=0}^{\infty}$ of disjoint closed subsets of Q such that H_n is homeomorphic to K^n. (So, H_0 is a one-point set, denote it by $\{t_0\}$.) For $n \in \mathbb{N}$, $k = 1, \dots, n$, let η_n be a homeomorphism $H_n \to K^n$, let $\pi_n^{(k)}$ be the canonical projection from K^n onto the kth coordinate space K and let

$$\varphi_n^{(k)} := \pi_n^{(k)} \circ \eta_n : H_n \to K. \tag{2.1}$$

Since K is uncountable, it has a subset Δ homeomorphic to the Cantor set $\prod_{n=1}^{\infty}\{0, 1\}$ (see [Ku], III.36.V, Corollary 1). By [P], Theorem 5.6, there exists a continuous surjection ϱ: $\Delta \to Q$ having a regular averaging operator. We denote by E (resp. E_n) the Borsuk–Kakutani extension operator $C(\Delta) \to C(K)$ (resp. $C(H_n) \to C(Q)$ (see [L–T], Theorem II.4.14.); since for $n \neq m$ the sets H_n and H_m have disjoint open neighborhoods, a simple trick shows that we may assume $E_n(f)E_m(g) = 0$ for $n \neq m$, $f \in C(H_n)$, $g \in C(H_m)$).

We now define

$$\psi(f) = E\varrho^\circ E_0 1 + \sum_{n=1}^{\infty} \prod_{k=1}^{n} E\varrho^\circ E_n \varphi_n^{(k)\circ} f. \tag{2.2}$$

(Here 1 denotes the constant function $H_0 \to \mathbb{C}$ with value 1.) If $\|f\| \leq c < 1$, we get the estimate

$$\left\| \prod_{k=1}^{n} E\varrho^\circ E_n \varphi_n^{(k)\circ} f \right\|_{C(K)} \leq c^n$$

which proves that ψ is a holomorphic mapping $U \to C(K)$.

We define the operator B_F. Let $\sum_n F_n$ be the Taylor series of F at 0 and let for all $n \in \mathbb{N}$ and $t \in K$ (resp. $t \in S$) $\mu(F,n,t) \in C(K^n)^*$ be as in Definition 1.1. Using (1.3) we see that for all t

$$(\|\mu(F,n,t)\|_{C(K^n)^*})_{n=1}^{\infty} \in \ell_1. \tag{2.3}$$

So we can define $\mu(F,t) \in C(Q)^*$ for all t by the convergent series

$$\langle f, \mu(F,t)\rangle = \sum_{n=1}^{\infty} \langle f \circ \eta_n^{-1}, \mu(F,n,t)\rangle. \tag{2.4}$$

We define $B_F \in L(C(K))$ (resp. $L(C(K), X^{**})$) by

$$(B_F f)(t) = \left\langle \varrho^{\circ -1} Rf, F_0(t)\delta(t_0) + \mu(F,t) \right\rangle \tag{2.5}$$

(resp.

$$\langle B_F f, t\rangle = \left\langle \varrho^{\circ -1} Rf, \langle F_0, t\rangle\delta(t_0) + \mu(F,t) \right\rangle \quad),$$

where $t \in K$ (resp. $t \in S$), $\varrho^{\circ -1}$: $C(\Delta) \to C(Q)$ is a contractive left inverse of ϱ° (see Section 1), $\delta(t_0) \in C(Q)^*$ is the point evaluation $g \mapsto g(t_0)$, $g \in C(Q)$ (recall $H_0 = \{t_0\}$ by definition) and R: $C(K) \to C(\Delta)$ is the restriction operator. Using (1.3) and (1.4) it is a straightforward matter to verify that B_F is a bounded linear operator between the given spaces.

We finally verify the identity $F(f) = B_F\psi(f)$. In the case $t \in K$, for $f \in U$,

$$(B_F\psi(f))(t) = \langle (\varrho^\circ)^{-1} R\big(E\varrho^\circ E_0 1 + \sum_{n=1}^{\infty}\prod_{k=1}^{n} E\varrho^\circ E_n \varphi_n^{(k)\circ} f\big), F_0(t)\delta(t_0) + \mu(F,t)\rangle$$

$$= \langle E_0 1 + \sum_{n=1}^{\infty}\prod_{k=1}^{n} E_n \varphi_n^{(k)\circ} f, F_0(t)\delta(t_0) + \mu(F,t)\rangle. \tag{2.6}$$

By the choice of E_n, $\operatorname{supp}(E_n g) \cap H_m = \emptyset$ for all $g \in C(H_n)$, $m \neq n$, hence (2.6) equals

$$\langle E_0 1, F_0(t)\delta(t_0)\rangle + \big\langle \sum_{n=1}^{\infty}\prod_{k=1}^{n} E_n\varphi_n^{(k)\circ} f, \mu(F,t)\big\rangle$$

$$= F_0(t) + \sum_{n=1}^{\infty} \big\langle \Big(\prod_{k=1}^{n} E_n \varphi_n^{(k)\circ} f\Big) \circ \eta_n^{-1}, \mu(F,n,t)\big\rangle$$

$$= F_0(t) + \sum_{n=1}^{\infty} \langle f\circ\pi_n^{(1)} \cdots f\circ \pi_n^{(n)}, \mu(F,n,t)\rangle = F(f)(t).$$

A similar calculation in the case $t \in X^*$ completes the proof of the existence of the universal mapping.

2° We prove the norm equality $||F||_{K,U} = ||B_F||$. First,

$$||B_F|| \le \sup_{\substack{||f||\le 1\\ t\in K}} \Big\{ |\langle \varrho^{\circ -1} Rf, \delta(t_0)\rangle F_0(t)| + \sum_{n=1}^{\infty} |\langle (\varrho^\circ)^{-1} Rf \circ \eta_n^{-1}, \mu(F,n,t)\rangle| \Big\}$$

$$\le \sup_{t\in K} \Big\{ |F_0(t)| + \sum_{n=1}^{\infty} \sup_{\substack{h\in C(K^n),\\ ||h||\le 1}} |\langle h, \mu(F,n,t)\rangle| \Big\} = ||F||_{K,U}.$$

For the converse, let $\varepsilon > 0$ and choose $s \in K$, $M \in \mathbb{N}$ such that

$$||F||_{K,U} < |F_0(s)| + \sum_{n=1}^{M} \sup_{\substack{h\in C(K^n),\\ ||h||\le 1}} |\langle h, \mu(F,n,s)\rangle| + \varepsilon/2. \tag{2.7}$$

For each $1 \le n \le M$ choose $f_n \in C(K^n)$ such that $||f_n|| \le 1$ and

$$\langle f_n, \mu(F,n,s)\rangle + \varepsilon 2^{-n-2} > ||\mu(F,n,s)||_{C(K^n)^*}. \tag{2.8}$$

We then have

$$||F||_{K,U} < |F_0(s)| + \sum_{n=1}^{M} \langle f_n, \mu(F,n,s)\rangle + \varepsilon. \tag{2.9}$$

Define $f := |F_0(s)|(F_0(s))^{-1}E\varrho^\circ E_0 1 + \sum_{n=1}^M E\varrho^\circ E_n\eta_n^\circ f_n \in C(K)$. The supports of the functions $E_n\eta_n^\circ f_n$ are disjoint for different n. Hence we have $||f|| = 1$. A calculation similar to (2.6) etc. shows that

$$(B_F f)(s) = |F_0(s)| + \sum_{n=1}^{M}\langle f_n, \mu(F, n, s)\rangle.$$

This and (2.9) imply $||B_F|| \geq ||F||_{K,U} - \varepsilon$.

The other case is similar. ∎

Let us denote by $\mathcal{K} \subset C(K)$ the cone of positive elements, i.e. functions f satisfying $f(t) \geq 0$ for all $t \in K$. Let $D \subset \mathcal{K}$ and let $F\colon D \to C(K)$. The mapping F is called **increasing**, if

$$F(f)(t) \leq F(g)(t) \tag{2.10}$$

for all $t \in K$, when $f, g \in C(K)$ are such that $f(t) \leq g(t)$ for all $t \in K$. Moreover, F is called **strictly increasing**, if it is increasing and a strict inequality holds in (2.10) for some t, if $f(s) < g(s)$ for some $s \in K$. (See [De], Section 19.3.)

We observe the following consequence of Theorem 2.1 and especially the definition (2.2): recall that the pull-back operators as well as the Borsuk–Kakutani extension operators are positive operators with respect to $\mathcal{K}$ (see [L–T], Theorem II.4.14.).

2.2 THEOREM: *Let K, U, X and S be as in Theorem 2.1; assume that X is reflexive. Every uniformly (K, U)-integral (resp. (S, U)-integral) holomorphic mapping $F\colon U \to C(K)$ (resp. $F\colon U \to X$) has a representation*

$$F = B_F \circ \psi, \tag{2.11}$$

where $B_F \in L(C(K))$ (resp. $B_F \in L(C(K), X)$), $\psi\colon U \to C(K)$ is holomorphic, $\psi(\mathcal{K} \cap U) \subset \mathcal{K}$, and the restriction of ψ to $\mathcal{K} \cap U$ is strictly increasing.

References

[A] R. Aron, *Holomorphy types for open subsets of a Banach space*, Studia Mathematica **45** (1973), 273–289.

[C] S.B. Chae, *Calculus and Holomorphy in Normed Spaces*, Marcel Dekker, New York, 1985.

[De] K. Deimling, *Non-linear Functional Analysis*, Springer-Verlag, Berlin–Heidelberg–New York–Tokyo, 1985.

[D–U] J. Diestel and J. Uhl, *Vector Measures*, American Mathematical Society, Providence, 1977.

[D1] S. Dineen, *Holomorphy types on Banach spaces*, Studia Mathematica **39** (1971), 241–288.

[D2] S. Dineen, *Complex Analysis in Locally Convex Spaces*, North-Holland Mathematics Studies, Vol. 57, North-Holland, Amsterdam, 1981.

[G–G–M] P. Galindo, D. Garcia and M. Maestre, *Holomorphic mappings of bounded type*, Journal of Mathematical Analysis and its Applications **166.1** (1992), 236–246.

[Kö] G. Köthe, *Topological Vector Spaces*, Vol. 2, Springer-Verlag, Berlin–Heidelberg–New York, 1979.

[Ku] K. Kuratowski, *Topology*, Vol. I, Academic Press, New York and London, 1966.

[L–T] J. Lindenstrauss and L. Tzafriri, *Classical Banach Spaces*, Lecture Notes in Mathematics **338**, Springer-Verlag, Berlin, 1973.

[Ma] P. Mazet, *Analytic Sets in Locally Convex Spaces*, North-Holland Mathematics Studies, Vol. 120, North-Holland, Amsterdam, 1986.

[Mu1] J. Mujica, *Linearization of bounded holomorphic mappings on Banach spaces*, Transactions of the American Mathematical Society **324** (1991), 867–887.

[Mu2] J. Mujica, *Linearization of holomorphic mappings of bounded type*, in *Progress in Functional Analysis* (K.D. Bierstedt, J. Bonet, J. Horvath and M. Maestre, eds.), North-Holland Mathematics Studies, Vol. 170, North-Holland, Amsterdam, 1992, pp. 149–162.

[Mu–N] J. Mujica and L. Nachbin, *Linearization of holomorphic mappings on locally convex spaces*, Journal de Mathématiques et Appliquées **71** (1992), 543–560.

[P] A. Pełczyński, *Linear extensions, linear averaging and application to linear topological classification of spaces of continuous functions*, Rozprawy Matematyczne **58** (1968), 1–89.

[R] R. A. Ryan, *Applications of topological tensor products to infinite dimensional holomorphy*, Ph.D. thesis, Trinity College, Dublin, 1980.

[S] M. Schottenloher, *ε-products and continuation of analytic mappings*, in *Analyse fonctionelle et applications* (L. Nachbin, ed.), Hermann, Paris, 1975, pp. 261–270.

[T1] J. Taskinen, *An application of averaging operators to multilinearity*, Mathematische Annalen **297** (1993), 567–572.

[T2] J. Taskinen, *Factorization of holomorphic mappings on $C(K)$-spaces*, preprint, 1994.